Symmetry energy of dilute warm nuclear matter

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The symmetry energy in the nuclear equation of state governs phenomena from the structure of exotic nuclei to astrophysical processes. The structure and the composition of neutron stars depend crucially on the density dependence of the symmetry energy [1]. As a general representation of the symmetry energy coefficient we use the definition

$$E_{sym}(n,T) = \frac{E(n,1,T) + E(n,-1,T)}{2} - E(n,0,T) \tag{1}$$

where $E(n;\ \delta;\ T)$ is the energy per nucleon of nuclear matter of N neutrons and Z protons with density n, asymmetry $\delta = (N\text{-}Z)/A$, and temperature T.

Our empirical knowledge of the symmetry energy near the saturation density, n_0, is based primarily on the binding energies of nuclei. The Bethe-Weizsäcker mass formula leads to values of about $E_{\mathrm{sym}}(n_0;\ 0)$ = 28-34 MeV for the symmetry energy at zero temperature and saturation density $n_0 \approx 0.16$ fm^{-3}, if surface asymmetry effects are properly taken into account. In contrast to the value of $E_{\mathrm{sym}}(n_0;\ 0)$, the variation of the symmetry energy with density and temperature is intensely debated. Many theoretical investigations have been performed to estimate the behavior of the symmetry energy as a function of n and T. Typically, quasiparticle approaches such as the Skyrme Hartree-Fock and relativistic mean field (RMF) models or Dirac-Brueckner Hartree-Fock (DBHF) calculations are used. In such calculations the symmetry energy tends to zero in the low-density limit for uniform matter. However, in accordance with the mass action law, cluster formation dominates the structure of low-density symmetric matter at low temperatures. Therefore, the symmetry energy in this low-temperature limit has to be equal to the binding energy per nucleon associated with the strong interaction of the most bound nuclear cluster. A single-nucleon quasiparticle approach cannot account for such structures. The correct low-density limit can be recovered only if the formation of clusters is properly taken into account.

TABLE I. Temperatures, densities, free and internal symmetry energies for different values of the surface velocity as derived from heavy-ion collisions (cols. 2-6), from the QS approach (cols. 7-8) and self consistently with clusters (cols. 9-12), see text.

V_{surf}	T	n	F_{sym}	S_{sym}^{NSE}	E_{sym}	F_{sym}^{QS}	E_{sym}^{QS}	T^{sc}	n^{sc}	F_{sym}^{sc}	E_{sym}^{sc}
(cm/ns)	(MeV)	(fm^{-3})	(MeV)	(k_B)	(MeV)	(MeV)	(MeV)	(MeV)	(fm^{-3})	(MeV)	(MeV)
0.75	3.31	0.00206	5.64	0.5513	7.465	6.607	8.011	3.26	0.00493	9.211	9.666
1.25	3.32	0.00165	6.07	0.5923	8.036	6.087	7.502	3.45	0.00511	9.295	9.647
1.75	3.61	0.00234	6.63	0.4137	8.124	6.877	7.896	3.54	0.00510	9.284	9.612
2.25	4.15	0.00378	7.81	0.1557	8.456	8.184	8.305	3.66	0.00495	9.193	9.524
2.75	4.71	0.00468	8.28	-0.0162	8.204	8.967	8.321	4.02	0.00510	9.274	9.386
3.25	5.27	0.00489	9.30	-0.1358	8.584	9.395	7.785	4.65	0.00574	9.683	9.227
3.75	6.24	0.00549	10.69	-0.2936	8.858	10.73	7.623	5.75	0.00684	10.49	8.978
4.25	7.54	0.00636	11.83	-0.4197	8.665	11.4	7.807	7.46	0.00866	11.98	8.964

In this work we employ a quantum statistical (QS) approach which includes cluster correlations in the medium. It interpolates between the exact low-density limit and the very successful RMF approaches near the saturation density. In this QS approach the cluster correlations are described in a generalized Beth- Uhlenbeck expansion. The advantage of this method is that the medium modifications of the clusters at finite density are taken into account. The method requires a sufficiently accurate model for the quasiparticle properties, for which we employ a RMF model with density dependent couplings which gives a good description both of nuclear matter around normal density and of ground state properties of nuclei across the nuclear chart. In order to extend the applicability of this RMF model to very low densities, it has been generalized in Ref. [2] to account also for cluster formation and dissolution.

Recently, the experimental determination of the symmetry energy at very low densities produced in heavy ion collisions of ^{64}Zn on ^{92}Mo and ^{197}Au at 35 MeV per nucleon has been reported [3]. Results of this study are given in the first four columns of Table I. The surface velocity V_{surf} , i.e. the velocity before the final Coulomb acceleration, was used as a measure of the time when the particles leave the source under different conditions of density and temperature. The yields of the light clusters $A \leq 4$ were determined as a function of V_{surf} . Temperatures were determined with the Albergo method [4] using a H-He thermometer based on the double yield ratio of deuterons, tritons, ^{3}He and ^{4}He, and are given in Table I as the average for the two reactions. The free neutron yield is obtained from the free proton yield and the yield ratio of ^{3}H to ^{3}He. To determine the asymmetry parameter of the sources the total proton and neutron yields including those bound in clusters are used. The proton chemical potential is derived from the yield ratio of ^{3}H to ^{4}He. The corresponding free proton and free neutron densities are calculated, and the total nucleon density is obtained by accounting also for the bound nucleons according to their respective yields [3]. The total nucleon densities are of the order of 1/100th to 1/20th of saturation density, as seen in Table. I.

An isoscaling analysis [5] has been employed (as a function of V_{surf}) to determine the free symmetry energy F_{sym} via the expression $\alpha = 4F_{sym}\ \Delta(Z/A)^2/T$. Here α is the isoscaling coefficient determined from yield ratios of Z = 1 ejectiles of the two reactions and $\Delta(Z/A)^2$ is the difference of the squared asymmetries of the sources in the two reactions. With $\Delta(Z/A)^2$ and the temperature determined as above, the free symmetry energy is extracted. From the free symmetry energy derived in this way from the measured yields, the internal symmetry energy can be calculated if the symmetry entropy is known. The values of the symmetry entropy $\mathrm{S}_{\mathrm{sym}}^{\mathrm{NSE}}$ for given parameters of temperature and density within the nuclear statistical equilibrium (NSE) model are shown in Table I, column 5. They are calculated with the equivalent expression of Eq. (1) as the difference between the entropies of pure proton or neutron and symmetric nuclear matter. The results obtained in this way for the internal symmetry energy $E_{\mathrm{sym}} = F_{\mathrm{sym}}+T\ \mathrm{S}_{\mathrm{sym}}^{\mathrm{NSE}}$ are shown in Table I, column 6. In Table I, we also give results of the QS model [2] for the free and internal symmetry energies (columns 7 and 8) at given T and n. There are large discrepancies between the measured values and the results of calculations in the mean-field approximation when cluster formation is neglected. On the other hand, the QS model results correspond nicely to the experimental data.

In Fig. 1 we present results for different approaches to extracting the internal symmetry energy and compare with the experimental values. In the left panel of the figure we show theoretical results for T at or close to zero. A widely used momentum-dependent parameterization of the symmetry energy (MDI) at temperature T = 0 MeV was given in Ref. [6] and is shown for different assumed values of the stiffness parameter x. For these parameterizations the symmetry energy vanishes in the low-density limit. We compare this to the QS result at T = 1 MeV. In this approach the symmetry energy is finite at low density. The T = 1 MeV curve will also approach zero at extremely low densities of the order of 10^{-5} fm^{-3} because the temperature is finite. The RMF, T = 0 curve is discussed below. Also note that the underlying RMF model for the quasiparticle description with n_0 = 0.149 fm^{-3}, $E_{\mathrm{sym}}(n_0)$ = 32.73 MeV gives a reasonable behavior at high density similar to the MDI, x=0 parameterization. We thus see that our approach successfully interpolates between the clustering phenomena at low density and a realistic description around normal density. In the right panel of Fig. 1 we compare to the experimental results, full circles (Tab. I, col. 6) in an expanded low-density region. Besides the MDI parameterization we show the QS results [2] for T=1, 4, and 8 MeV, which are in the range of the temperatures in the experiment. The QS results including cluster formation agree well with the experimental data points, as seen in detail in Fig. 1. We conclude that medium-dependent cluster formation has to be considered in theoretical models to obtain the low-density dependence of the symmetry energy that is observed in experiments.

The temperatures and densities of columns 2 and 3 in Table I will be modified if medium effects on the light clusters are taken into account [7]. We have carried out a self-consistent determination of the temperatures T^{sc} and densities n^{sc} taking into account the medium dependent quasiparticle energies as specified in Ref. [8] (columns 9 and 10 of Table I). Compared to the Albergo method results [3], the temperatures T^{sc} are about 10% lower. Significantly higher values are obtained for the inferred densities n^{sc} which are more sensitive to the inclusion of medium effects. We have also calculated the free and internal symmetry energies corresponding to these self-consistent values of T^{sc} and n^{sc} according to Ref. [2] (columns 11 and 12 of Table I). These results are also shown in the right panel of Fig. 1 as open

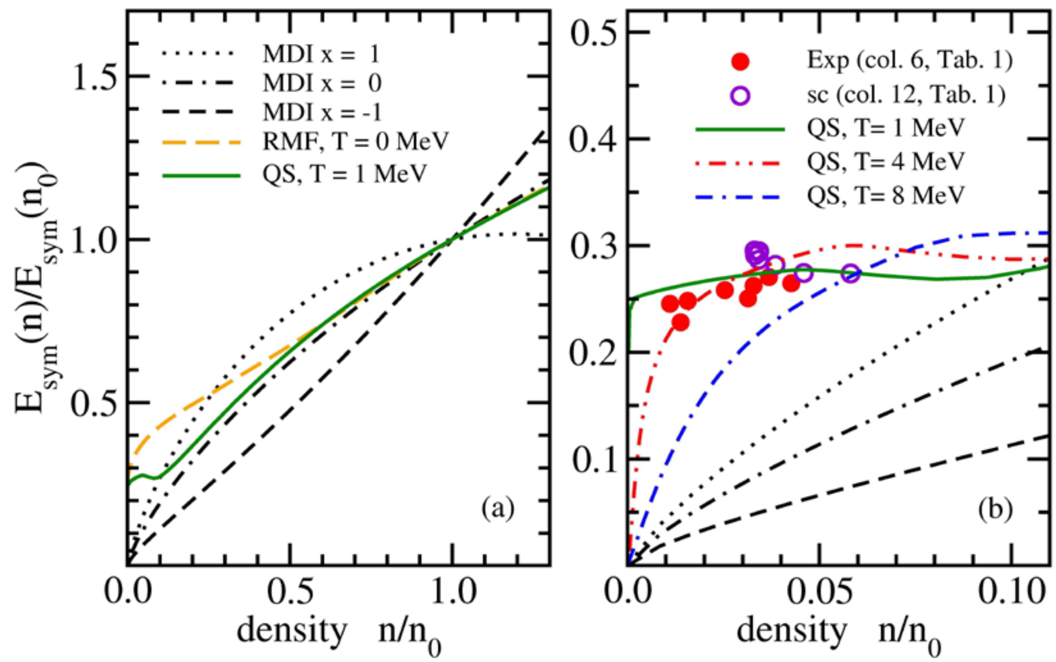

FIG. 1. Comparisons of the scaled internal symmetry energy $E_{sym}(n)/E_{sym}(n_0)$ as a function of the scaled total density n/n_0 for different approaches and the experiment. Left panel: The symmetry energies for the commonly used MDI parameterization for T = 0 and different asy-stiffnesses, controlled by the parameter x (dotted, dot-dashed and dashed lines); for the QS model including light clusters for temperature T = 1 MeV (solid line), and for the RMF model at T = 0 including heavy clusters (long-dashed line). Right panel: The internal scaled symmetry energy in an expanded low density region. Shown are again the MDI curves and the QS results for T = 1, 4, and 8 MeV compared to the experimental data with the NSE entropy (solid circles) and the results of the self-consistent calculation (open circles) from Table I.

circles. The resultant internal symmetry energies are 15 to 20% higher than the QS model values for T and n given in columns 2 and 3 in Table I.

We have restricted our present work to that region of the phase diagram where heavier clusters with $A > 4$ are not relevant. The simplest approach to model the formation of heavy clusters is to perform inhomogeneous mean-field calculations in the Thomas-Fermi approximation assuming spherical Wigner-Seitz cells. In Fig. 1 (left panel) preliminary results for the zero-temperature symmetry energy of such a calculation is shown by the long-dashed line using the same RMF parameterization as for the QS approach introduced above; for details see Ref. [9]. The symmetry energy in this model approaches a finite value at zero density in contrast to the behavior of the MDI parameterizations and conventional single-nucleon quasiparticle descriptions.

In conclusion, we have shown [10] that a quantum statistical model of nuclear matter, that includes the formation of clusters at densities below nuclear saturation, describes quite well the low-density symmetry energy which was extracted from the analysis of heavy-ion collisions. Within such a theoretical approach the composition and the thermodynamic quantities of nuclear matter can be modeled in a large region of densities, temperatures and asymmetries that are required, e.g., in supernova simulations.

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